

On the Origin of the 60 Base and the 21600' of a Circle

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Abstract

Present paper is an attempt to understand the origin of the sexagesimal number system regarded as an enigma by modern historians of Mathematics. A review is presented of the various explanations existing along with discussion as to what has prevented a consensus in the matter. As a better and more convincing theory it is proposed that the sexagesimal system arose out of the practice of counting breaths employed to keep a measure of time. It is suggested that the time structure apparent in breathing led the ancient sages to conceive the sexagesimal system and the ecliptic accordingly got divided into 21600, the number of breaths in a day. Attestation to the above is shown as available in Sūryasiddhānta and Āryabhatīya which describe the diurnal rotation of Earth in terms of 21600 Prana.

Sexagesimal notation also facilitated the fixing of precession effect in terms of the mystic numbers as 50/60 minute in a year and with the aid of astronomical observations various solar year lengths and the length of lunation could be calculated precisely. It is likely that the accuracy of Babylonian time keeping and astronomical observations arose out of their measurement of time in terms of the number of breaths.

Key Words:

Sexagesimal system, Number, 21600, Breath, Babylonian, Sumerian, Sūryasiddhānta, Āryabhatīy, Prāṇa, Yoga, Precession

Introduction

The phenomenal basis of the ancient world conceptions underlying astronomy and mathematics had been a topic of interest to modern society and much headway could be made in this direction by researchers during the last 200 years. Babylonian, Greek and the Indian theatres of ancient wisdom had been under detailed examination by both western and indigenous scholars to understand the origin and evolution of the basic conceptions we have inherited through the knowledge corpus of astronomy and mathematics. To spell out a marvelous few with which we are accustomed –

1. Decimal Number System
2. Sexagesimal Number System
3. Division of the Circle into 60 based units like 360 and 21600'
4. Division of the Day into 24 hours and also into 60 based Units
5. Weekdays numbering seven

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All these conceptions can be traced to mathematical cuneiform texts of the Old Babylonian period viz. 1900 – 1650 BCE and in later times in the Greek and Indian texts of Astronomy. Most important of the above are the sexagesimal base and the associated divisions of time (Earth's rotation) and Ecliptic into 60 based Units which have defied the comprehension of the modern minds. In the history of Mathematics, the sexagesimal number system is ascribed to Sumerians but the reasons which led to such a choice remain an enigma to date.

Review of the Explanations¹

1. Divisors of 60 and the resultant ease in arithmetical applications

Theon of Alexandria (AD400), J.Wallis (AD 1700) and E. Lofler (AD 1910) have been proponents of this thesis at different epochs of history. This explanation demanded recognition of 60 as the lowest number having maximum divisors and the set of first six integers among the divisors contributed to ease in application. 60 has altogether 12 subdivisors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 but we do not find the use of 60 in the area of weights and measures where the role of divisors is more significant. We see the use of 12 as a base for weights and measures frequently because of the divisors 1, 2, 3, 4, 6 and 12. Further, we need to keep in mind that the weights and measures of which records came to exist in ancient society may have been the output of a number system already in vogue.

2. Derivation from the count of 360 days

Formaleoni in 1788 and M. Cantor in 1880 forwarded the explanation that the count of 360 days had been the natural cause of division of the circle into 360 degrees and the division of a circle into sextants having chords equal to radius led to the choice of 60 as a unit of reckoning.

This explanation is far too theoretical and scholarly given the fact that the normal development of a number system like the counting process has to be from lower to higher units. So it is quite unlikely that the smaller unit of 60 evolved out of the larger count of 360 days which for Babylonians might have been fictitious as they knew the year to be more than 360 days. For Babylonians the number of days in a year to be taken as 360 demanded alternate significance for 360.

In recent times astronomical explanations have been constructed based on the return of Jupiter and Saturn which takes a period of 60 years to explain the origin of the sexagesimal system vis-a-vis the significance of base 60. It is quite unlikely that the Babylonians sat down to deliberate as with modern times under the banner of a committee and they choose the 60 based on the astronomical records of hundreds of years. If we accept that the decimal system arose out of the fingers on hands and the primitive counting it is likely that the base 60 too had its origin from a quite familiar daily experience than the experience of heavens over hundreds of years. In fact without a number system and significance to numbers like 60 the experience of the heavens would not have received much attention to the early astronomers. Correspondence that cosmic

bodies stuck with numbers may have been one of the first experience of mystery and mystic correspondence that the Sumerians had.

3. Origin of base 60 from systems of weights and measures

O. Neugebauer proposed that the decimal number system when combined with the demands of fractions like $\frac{1}{2}$ and $\frac{1}{3}$ in weights and measures led to the derivation of a new system based on 60. Thesis did not gain recognition as the base 60 is found only in Sumer even though the weights and measures factor had been universal. Convenient expression of fractions in the sexagesimal notation is illustrated below in table.

$\frac{1}{2}$	=	0;30	$\frac{1}{15}$	=	0;4
$\frac{1}{3}$	=	0;20	$\frac{1}{16}$	=	0;3,45
$\frac{1}{4}$	=	0;15	$\frac{1}{18}$	=	0;3,20
$\frac{1}{5}$	=	0;12	$\frac{1}{20}$	=	0;3
$\frac{1}{6}$	=	0;10	$\frac{1}{30}$	=	0;2
$\frac{1}{8}$	=	0;7,30	$\frac{1}{40}$	=	0;1,30
$\frac{1}{9}$	=	0;6,40	$\frac{1}{50}$	=	0;1,12
$\frac{1}{10}$	=	0;6	$\frac{1}{1:00}$	=	0;1
$\frac{1}{12}$	=	0;5			

Even though the convenient use of 60 in expressing fractions is unique it is unlikely that Sumerians made 60 as the base with a table of the above kind in their thinking. Possibility of base 12 given the shadows we see of duodecimal units like 12 and 24 hours that we see in cuneiform records makes the above proposition untenable.

4. From 30 to 60 and Geometrical Considerations

E. Hope explained that normally 30 would have met the needs but the Sumerians chose 60 as it was divisible also by 4. He also claimed that if the sextant of a circle was configured in terms of 10 ‘degrees’ as per the decimal system, the circle had to be 60 and thus the choice of 60.

This explanation for the base 60 needs the base of 10 for survival and is thus not complete in itself and also assumes much geometry and abstraction in the development.

5. Fusion of two counting systems and civilizations

G. Kewitsch (AD1904) suggested the fusion of a 10-based and 6-based counting as a result of the fusion of two civilizations. But the theory could not receive acceptance as a 6-based counting is not known anywhere in the world. Scholars have also pointed out the possible origin from a 12-based duodecimal system or from religious and mystic significance of the number 60.

Origin of the decimal system from finger counting has inspired scholars to forward an explanation involving the count of phalanges on fingers which could have resulted in a count of 60 with some unique practice.

6. Mystical significance of numbers

It is well attested that numbers played an important role in the Mesopotamian conceptions of the structure of the Universe. In many respects the Sumerian conceptions of 2nd Millennium BC meets a parallel in the later Indian notions of time and cosmos. To cite a few:²

- (a) Šaros = 3600 represented the Cosmos and one complete rotation of Earth on its axis. We can see the Indian time notion of the division of a day into 60 *Ghatis* and 3600 *Vighatis* as reflective of the Mesopotamian notion.
- (b) Mesopotamian conceptions of Ki-Šar and An-Šar, the lower and upper worlds or microcosm and macrocosm have their parallel in the Indian notions of Piṇḍāṇdam and Brahmāṇdam. Number 60 (Geš) and 1/360 of a circle, both have the same name in Mesopotamian terminology and the symbol of Geš is Phallus and the sign also represented Man and the masculine upper world while number 50 represented Earth or the feminine lower world. We can see corresponding speculations in Tantra where Siva and Sakti signify the mating upper and lower worlds from which the Gods are born. Number 60 is therefore credited to the supreme God 'Anu' and so its acceptance as the base for the number system.

Most of these explanations are theoretical and presupposes geometry, world, and mystic etc conceptions before the adoption of 60 as the base of the number system. As for example nothing prevented the Babylonians from crediting 60 to God 'Anu' as 60 was important as the base of the number system. This proposition is more plausible than 60 having mystical significance and association with Anu before the origin of the sexagesimal system. It is widely accepted that our efforts to understand the past is prone to the danger of imposing the present on the past and thus utmost caution is called for in the efforts to understand and interpret the ancient conceptions.

60-base Notions in Indian Astronomy

In explaining the Mesopotamian origin of the sexagesimal system, we can find some quite valuable information preserved in Indian astronomical texts like *Sūryasiddhānta*³ and *Āryabhaṭīya*⁴ which have been dated to 5th century AD. Indian borrowing from Mesopotamian astronomy had been the subject of serious researches in history of astronomy and stands universally accepted. Against such background of a Mesopotamian legacy of the Indian astronomical notions, we see in *Sūryasiddhānta* –

प्राणादिः कथितो मूर्त्तः त्रुट्याद्योऽमूर्त्तसंज्ञकः
षड्भिःप्राणैर्विनाडीस्यात् तत्षष्ट्यानाडिकारम्भता (11)

Time in units of prāṇa is real and experienced and those like truṭi are imaginary. Six prāṇas make a Vināḍī, sixty of which is one nāḍī.

Also, we can find in *Gītikā* 6 of *Āryabhaṭīya* - प्राणैककलां भूः i.e., the earth rotates one minute of arc in one breath. Pauliśa *siddhānta* too aired the same notion as is evident from *Alberuni*⁵.

If we extend the above notion to one diurnal rotation of Earth, we can see that 21600 minutes of arc had its origin from the number of breaths in a day. A day had also units in terms of 60s like the 60 Ghatikas and 3600 Vighatikas with the Ghatikā made up of 360 and Vighatikā of 6 breaths. Obviously, one degree rotation of Earth ($1/360$) was equivalent to 60 breaths and we find this experience or observation reflected in the Mesopotamian notion cited above i.e. *Number 60 (Geš) and $1/360$ of a circle, both have the same name in Mesopotamian terminology and the symbol of Geš is Phallus and the sign also represented Man and the masculine upper world An-Šar.*

It is also well attested in cuneiform texts of the Seleucid era that the day had a division into 360 units called uš (meaning originally length) and thus each uš had to be of the duration of 60 breaths.⁶

Thus the correlation that we see in Mesopotamian notions of the Number 60 and $1/360$ of a circle or 1^0 rotation of Earth arose out of the 60 breaths during the interval. This observation leads us to a new theory on the origin of the sexagesimal system as well as the ancient astronomical conceptions like 21600 arc minutes of a circle as arising from the human experience and observation as well as count of breathing as a measure of time.

Evidences for the ‘Breath Number’ theory

We can find a number of supporting facts in other Mesopotamian astronomical conceptions which support the above Breath Number theory of the origin of sexagesimal system.

First and most fundamental aspect is the practice of 'body counting' which gave one-to-one correspondence for numbers with fingers and other body parts.⁷ Such practices are well attested in all primitive societies. *Given such body origin of counting and numbers, it is quite likely that the count of breaths became a tool for the time reckoning by means of counting in the ancient civilizations.*

Breathing as the sign of life obviously may have at the centre of attention for the ancient occult schools and the count of breaths as such may have been the best time machine available to them.

Breath gave them the smallest possible unit of time which they could have measured and the count could have easily led them to bigger units like 360 counts in 24 minutes or $1/60^{\text{th}}$ of a day.

The fact that no other time measurement like sun-dial or water clock or flower clocks could have been more efficient than breathing also explains the use of sexagesimal system in astronomy where all observations had to be associated with time.

Existence of mystic thought among Sumerians also suggests the possibility of breathing exercises like Prāṇāyāma in antiquity and thus the number of breaths as a measure of time based on '60' may have been a significant factor that inspired abstract notions about numbers. One of the six Indian schools of Philosophy (Darśanam) viz., Sāmkhya,⁸ owes its name to numbers or enumeration. Sāmkhya is well known as an abstractive effort and is often construed as the theoretical side of Yoga to which Prāṇāyama is a part. The fact

that Sāmkhya had its origin out of the sphere of Vedic influence suggests the possibility of the doctrine as a development of the Dravidian or Sumerian mysticism which before the advent of the Āryans extended over the vast geographical areas between Egypt and India. Mystical significance to numbers arose out of the intellectual efforts to understand the phenomena of life in terms of its origin as well as evolution and we can see such trends in the development of the various schools of mysticism in India. Even though now known only through the Sanskrit texts, the Indian schools of Tantra are perhaps the nearest approximations that we can find for the ancient Sumerian schools of mysticism. It is quite likely that mysticism and notions like weekdays, hours and zodiacal symbolism etc which became universal in later times had their origin in the cultural unity of Indo-Mediterranean proto-history.⁹

Astronomical Evidence

1. Hitherto we have been explaining the 360 degrees as arising from a day count rounded off to 360, a multiple of 60. A better explanation shall be the origin of 60(*Geṣ*) and 360 from the diurnal rotation and breathing and evolution of the day count of 360 from the same. But the Solar return demanded completion of the 360⁰ sojourn and the evolution of solar year.
2. An independent observational origin of 60 as a unit from the count of breaths, also explains the Unit of 30 lunar days for the synodic circuit of Moon. It is important to note here that Indian astronomy even today carry the vestige of a half-lunar day called '*Karana*' which perhaps reflect the ancient conception of lunar circuit in terms of the base 60.
3. Breath Number theory also renders satisfactory explanation to the Saros, 3600 years (60x 60) in which the precession amounted to 50 degrees. 60² Years = 50⁰ rotation of Earth's axis reflected the mystical importance of number 60 as credited to Anu and 50 to Earth. Degrees that remained in a quadrant $90 - 50 = 40$ got credited to the God of netherworld. 21600 minutes of arc also gave rise to the higher number 1296000" and its fractions like 432000, 864000 etc which are well attested in Mesopotamian records as well as in Indian astronomical and Puranic texts.
4. Division of day based on breaths also explains the origin of the 1hour/1080 unit available in Babylonian cuneiform texts. 21600' or 1296000" divided by 24 hours gives 54000" = 1 hour and 1 hour/1080 = 50" seconds of arc. Origin of this unit can be explained in terms of precession and also the Babylonian units of length še and shu-si (*yavam* and *angulam* in Indian texts).
5. In terms of breaths, the tropical and sidereal year differed by $50/60 = 0.8333$ equivalent to 50" of arc and this factor alone shall provide explanation for the Mesopotamian mysticism associated with numbers. Precession through 21600 breath units therefore required $21600/0.8333 = 25920$ years. It must be noted here that 60 was the number of Man and thus represented breath while 50 was the number of Earth and thus represented the seasons.

It becomes apparent that the number of breaths equal to 21600 and the sexagesimal system facilitated the cognizance of precession in terms of 50/60 minutes and the sidereal length of the year could be computed simply using the mystic numbers from the length of the well known Metonic cycle.

19 years = 235 lunations gave the year as $6939.688/19 = 365.24675$

$365.24675 + (50/60)/60 = 365.26063$ which is the length of the Babylonian sidereal year traceable to the cuneiform records (the period relation 3;45 years = 46;23 synodic months i.e. $225^y = 2783^m$ ¹⁰)

Further, the above anomalistic value could be used for deriving the tropical year length as $365.26063 - (60/3600) = 365.2439$ or 365.259 giving 365.2423 .

Sexagesimal system and the numbers 50 and 60 thus rendered a cosmic experience in numbers.

6. Explicit mention of the length of any solar year cannot be found in Cuneiform texts. But Al-Birunī has described the value of 365;15, 30 as Babylonian and this value matches well with the Indian Siddhāntic value of 365.25875 days (365;15,31,30). When compared with the seasonal year this value is in excess by 358 breaths ≈ 360 and thus a shift of 1 seasonal day in 60 years and year in 21600 years. Further, if we look at the Babylonian System A lunar theory we can understand that the period relation 3;45 years = 46;23 synodic months ($225y = 2783m$) can be true only for an anomalistic year that exceeds the seasonal by 360 breaths or 6 uš in a year¹¹. We must note here that the number of years 3;45 = 225 is specifically chosen to have a shift of 3.75 degrees just as 3600 years = 60;0^y causes a shift of 60 degrees when the year is in excess of seasonal year by 360 breaths or 6 uš in a year.

Babylonian cuneiform records have evidence for their knowledge of the Metonic cycle of 19 years ($19y = 3;55^m$) which leads to an year of 12;22,6,20 instead of the greater value of 12:22, 8 synodic months and thus the latter choice of a larger value must be inspired by special reasons.

7. Even though scholars have rejected the possibility of a 6-based system as the cause of the 60-base, it can be seen that the number 6 had been playing very important role in Babylonian astronomy. Lunar theory considered the latitudes as $\pm 6^0$ and the Zodiac was conceived as a belt of 12^0 width and each lunar day amounted to 12^0 .

Evidence from measurements of Lengths

Above derivations of numbers and the 50/60 (=5/6) relationship finds support in the Babylonian units of lengths adapted to angular measurements.

- (a) Ancient units of length such as barleycorn (še), known in Indian texts as 'yavam'

6 še or Yavam = 1 shu-si (Angulam or Ring-finger)

7.2 Yavams = 1 Thumb
 1 Ring-finger = $5/6 = 0.8333$ of Thumb
 60 Ring-fingers = 50 Thumbs
 50 Thumbs = 360 Yavams (še)
 1 finger = 5 mins of arc
 4 fingers = 1 Palm
 1 cubit = 30 fingers = 2.5^0
 12 cubit = 1 sign
 12 sign = 360^0

- (b) Considering the application of the above units in astronomy, $1^0 = 12$ fingers = 72 še or yavam and $360^0 = 25920$ še = 4320 shu-si (*angulam*) and thus the hours credited to Horus had 1080 še units of rotation per hour. In the absence of the number $25920 = 24 \times 1080$, there is no logic in conceiving a unit like 1080 še in 1 hour.
- (c) Quadrant = $90^0 = 18$ Units is also attested in antiquity¹² and with $1^0 = 12$ fingers = 72 še, we get a quadrant as $90^0 = 90 \times 12 = 1080$ fingers = $18 \times 60 = 36 \times 30 = 36$ cubits.
- (d) We can thus see the shadows of a 6-based, 12-based and 60-based number system emerging as a result of the measurement of time interval in terms of the number of breaths which made up to a total of $21600 = 5 \times 6 \times 12 \times 60$ in a day. $5 \times 60 = 30$ and $5 \times 6 \times 12 = 360$ were implicit in 21600 as to become later the lunar day count and also to be the solar day count in a year.

It is apparent therefore that the two systems of unit viz., the count of breaths or *prāna* for the measure of time and the *yavam* or *angulam* for the measurement had given rise to the mystic properties of the base numbers of Mesopotamia.

- (e) As noted by Neugebauer, the inaccuracy of time measurements using sundials or water clocks had no noticeable influence on the great accuracies achieved in Babylonian astronomy. Such progress was made possible by the concept of time-degrees i.e. rotation of Earth in 360 units called *uš* consisting of 60 breaths. Unit of *uš* based on the diurnal rotation of the Earth is another pointer towards the original of the sexagesimal system and the 21600' of arc from the number of breaths as we find recorded in Indian astronomical treatises.

Units and the Astronomical Phenomena

Above units of breaths and fingers display interesting correlations to the astronomical phenomena and parameters like the sidereal year, tropical year and anomalistic year which are well known in Mesopotamian astronomy. sidereal and tropical years differed by 300 breaths while anomalistic and tropical years differed by 360 breaths to make the difference between anomalistic and sidereal years equal to 60 breaths. 1 day or 21600 breaths differed between sidereal and tropical years in 72 years and with anomalistic year the 1 day difference occurred in 60 years as we see in Siddhāntic astronomy. Considering for example the modern year lengths:

Sidereal year = 365.2563624 days
Tropical year = 365.24219 days
Difference = 0.01417 days = 306 breaths
Siddhāntic year = 365.25875 days
Difference with modern tropical = 358
Difference with modern sidereal year = 52

The Babylonian difference of 300 breaths between sidereal and tropical years differed only by 6 breaths or 24 secs in a year when compared to the difference in modern values.

Scholars like O. Neugebauer¹³ have opined that the precession of equinoxes came to be discovered around the time of Hipparchus and thus the longitudinal measurements came to be effected with vernal equinox fixed as the zero point^f. Given the fact that the 8⁰ and 10⁰ norms are very popular in cuneiform texts corresponding to System B and System A, it is likely that precession was known to Babylonians and also they had arithmetical methods as above to compute and refix the cardinal points with the passage of time. This is somewhat obvious from the fact that the observable year was tropical or seasonal and had been known to them in terms of the eclipse cycle Saros and related 19 years and 235 lunations. Acceptance of longer periods like 225 years in luni-solar theory with years having the anomalistic value could have been possible as a result of some arithmetical methods with which they may have been accounting for precession. Babylonian period relations for Planets with large periods like 46, 1151, 284, 427, 265 also suggests that Babylonians computed planets relative to the shifting cardinal points. Various norms of Aries 8, 10, 12, 15 etc are attested in literature and in Neugebauer¹⁴ an account of the same is available.

Correlation to Yoga precepts on Prāṇa

Discussion given above on the Mesopotamian units of time and length bear some interesting correlation to the yoga precepts on Kāla and Prāṇa. What we see in Sanskrit texts of medieval times can be the compilation of ancient wisdom in Sanskrit as the Tantra and Yoga schools became more popular in India. Svachanda Tantra described the phenomenon of time and breathing in the following words:

1. कालो द्विधात्र विज्जेयः सौरश्चाध्यात्मिकः प्रिये¹⁵ – Commentary by Kshemarāja explains that ‘Time even though one only it is experienced as the external, macrocosmic solar time and the internal or microcosmic time experienced through the successive breaths or prāṇa.
2. प्राणापानमयः प्राणो विसर्गापूरणं प्रति¹⁶ – Breath as the unit Prāṇa is described in terms of exhalation and inhalation which are known by the names ‘prāṇa’ and ‘apāna’.

^f “The reckoning in the Almagest of the 360 degrees of longitude, beginning at the vernal equinox, called Aries 0⁰, is of course, related to discovery of precession and the resulting decision to define the solar “year” as the tropical year”

3. Units of time as enumerated and explained in this work is typical of the Tantra doctrines –

मानुषाक्षिनिमेषस्य अष्टमांशः क्षणः स्मृतः
क्षणद्वयं तुटिरज्ञेया तद्द्वयं तु लवः स्मृतः (11-201)
लवद्वयं निमेषस्तु ज्ञातव्यो गणितक्रमात्
दशपञ्च निमेषाश्च काष्ठा चैव प्रकीर्तिता (11-202)
त्रिंशत् काष्ठाः कलाः ज्ञेया मुहूर्त्त्रिंशदेव ताः
मुहूर्त्स्तु पुनःस्त्रिंशदहोरात्रस्तु मानुषः (11-203)¹⁷

Here we see the minutest unit of time as Kṣaṇa = 1/8th of an eye-lash, Tuṭi = 1/4th of an eye-lash, Lavam = 1/2th of an eye-lash, Nimeṣam = eye-lash = 2 Lavam, 15 Nimeṣam = Kāsthā, 30 Kāsthā = 1 Kalā, 30 Kalā = 1Muhūrttam, 30 Muhūrttam = 1 Ahorātram. i.e. 1 Ahorātram = 900 Kalā = 27000 Kāsthā and 1 Kalā = 4 Vighaṭis, 15 Kalā = 1 Ghaṭi, 3600 Vighaṭis = 1 Ahorātram = 60 Ghaṭis.

4. Interrelationship of the external and internal times are described as –

तुटयः षोडश प्राणे पूर्वं हि कथिता मया
बाह्येनैव तु कालेन ते लवाः परिकीर्तिताः (7-27)

ताभिश्चतसृभिः देवि प्राणो यामो विधीयते
तैरेव प्रहरैर्देवि चतुर्भिस्तु दिनं भवेत् (7-28)

आध्यात्मिकाहोरात्रेण बाह्ये काष्ठा विधीयते
मासेनाध्यात्मिकेनैव बाह्ये चैव कला भवेत् (7-50)

तत्र त्रिंशदहोरात्रा मासस्तु वरवर्णिनी
मासैर्द्वादशभिश्चैव बाह्ये थ घटिका भवेत् (7-51)¹⁸

16 Tuṭis of Prāṇa which is 2 Lavam of Prāṇa is 1 Lavam of Sūrya. 4 of these or 8 Lavams make 1 Yāmam of Prāṇa and 4 Yāmams constitute the Day of Prāṇa (and equal is the Night). 64 Lavams or 32 Nimeṣams of Prāṇa thus make 1 Kāsthā of Sūrya or 1 Ahorātra of Prāṇa. 1 Nimeṣam of Prāṇa is thus apparently 1/10 of the second of external time. External day and night are thus 432000 Nimeṣams of Prāṇa to yield the Ahorātram as 864000 Nimeṣam of Prāṇa.

Internal year is equated to be the solar Ghaṭikā = 24 mins = 1440s = 12 months = 360 breaths. 1 month = 120 secs. 1 day = 4 secs. 1 Ahorātra of the Prāṇa thus gets equated to a breath or unit of Prāṇa.

5. Further the external (bāhya) Ghaṭikā is equated to internal (ānthara) year.

घटिकाः षष्टिस्त्वहोरात्रे बाह्ये तु प्रवहन्ति वै
ता एवान्तरचारेण षष्टिः संवत्सराः स्मृताः (7-53)

Here we see the experience of 60 in the breathing phenomenon and also 360 appear as the number of breaths in 1 external Ghaṭikā.

6. Number of breaths in a Day

षट्शतानिवरारोहे सहस्राण्येकविंशतिः

अहोरात्रेण बाह्येन अध्यात्मं तु सुराधिषे (7-54)

Verse gives the number of breaths as $600 + 21000 = 21600$ which is the minutes of arc through which the Earth rotates.

7. Measurement of Space in Breathing

हृदयोर्ध्वे तु कण्ठाधो यावद्वै प्रवहेत् प्रिये
अङ्गुलेन विहीने तु प्रथमः प्रहरः स्मृतः (7-33)

द्वितीय ऊर्ध्वे विज्ञेयो मध्याह्नस्ताल्दमध्यतः
अत्र होमो जपो ध्यानं कृतं वै मोक्षदं भवेत् (7-34)

नासाग्रत्रय्यङ्गुलोर्ध्वे तु यावत्प्राप्तस्तु सुव्रते
प्रहरस्तु तृतीयो सौ भवेद् वै वरवर्णिनी (7-35)

चतुर्थान्ते तु देवेशी प्राणसूर्यः सदास्तगः -

In these verses we can find the ancient wisdom which equated the breath with the diurnal rotation. The path of prānārka or prāna-sūrya had 4 divisions in exhalation with the 9 aṅgulas from the heart to the throat-end and 9 aṅgulas from throat-end to the zenith (tālu-madhyam) – at the end of this 18 aṅgulas is the mid-day for prāna. Then from zenith to the nose-tip via forehead 6 aṅgulas and from nose-tip to dvādaśāntham makes 12 aṅgulas of the total of 36 aṅgulas that make up the exhalation. At 12 aṅgulas below the nose-tip sunset is conceived and the smallest unit of time Tuṭi is the time lapse between the exhalation and inhalation.

Prāna or breath is thus measured in units of angula and 72 angulas make 360 degrees. Thus 432 yavams make 360 degrees or $1^0 = 1.2$ yavam or 1 yavam = $50'$ = 50 breaths. 1 angulam = 5 degrees = 300 breaths or prāna.

Air flow gets divided into 6 stages of 12 aṅgulams each and the origin of Prāna is at 12 aṅgulams below the nose-tip and is named as 'dvādaśāntham'.

8. Association with sidereal Zodiacal Signs

हृदपत्मादूर्ध्वपर्यन्तं राशयः षड् व्यवस्थिताः
अङ्गुलैः षड्भरेकैको हृदपत्माद्याव शक्तितः (7-93)

हृदयादुदयस्थानात्संक्रान्तिर्मकरे स्थिता
षडङ्गुलान्यधस्त्यक्त्वा कुंभे संक्रमते पुनः (7-94)

गलोर्ध्वाद्यावत्ताल्वन्तं त्यक्त्वा मेषेथ संक्रमेत्
नासान्तं यावत्संक्रान्तिरङ्गुलानि षडेव हि (7-95)

नासाग्रं तु परित्यज्य प्राणहंसो वृषे चरेत्
षडङ्गुलानि संत्यज्य संक्रमे मिथुने पुनः (7-97)

These precepts and the continuing discussion in Svacchanda tantra in verses 112-114¹⁹ place the 12 zodiacal signs in the path of breath – 6 aṅgulams of the outgoing breath constitute the signs respectively from Makarādi (sidereal Capricorn⁰) which is

‘Hrdaya-kamlam’ of the Yogis and the incoming breath begins at Karkkādi (sidereal Cancer⁰) which is named as ‘śaktyantham’ or ‘dvādaśāntham’.

Outgoing breath is termed as *Uttarāyana* or the Northern course while the incoming breath is termed as *Dakṣiṇāyana* or the Southern course. *Uttarāyana* of Yoga is therefore always from *Makarādi* to *Karkkādi* and *Dakṣiṇāyana* from *Karkkādi* to *Makarādi*. Midpoints of the outgoing and incoming halves of breath are termed as Viṣuvams and correspond to the sidereal Aries⁰ and Libra⁰. These peculiar yoga conceptions quoted in Indian astronomical texts have caused lot of confusion in understanding them in relation to the tropical signs.

This occult literature dealing with Japayoga and the experience of the ‘mantra-tattva’ is replete with the use of 6 and 12 enunciating the philosophy and practice of Japa. As for example the Indian notion of time is based on units called ‘akṣara’ or syllable –time taken to have a syllable uttered.

10 Gurvakṣara) = 1 Prāṇa

6 Prāṇa = 1 Vinādikā

60 Vinādikās = 1 Nādikā

60 Nādikās = 1 Day

Not only that the later Sanskrit texts give Zodiacal or astronomical symbolism but also we can see that the astronomical and time units Vinādikā, Nādikā in works like Sūryasiddhānta which predates by 1000s of years have their origin from the term ‘nādi’ which refers to the channel of breath in the human body. It can be found that –

यथा पर्णं पलाशस्य व्याप्तं सर्वत्र तन्तुभिः
शरीरं सर्वजन्तूनां तद्वद्व्याप्तं तु नाडिभिः (7-97)

Further the precepts are based on the correspondence between the Zodiac and the physical body which we can trace to the Sumerian occultism.

We can find a real demonstration of the 50/60 correlation that the count of breaths vis-a-vis sexagesimal system bears to the experience of the Cosmos via precession, the difference between the sidereal and tropical year. Number of breathings and the sexagesimal system rendered an obvious computation of the annual rate of precession amounting to 50" = 50/60 minutes of arc = 300/360 = 21600/25920 where 25920 is the mean value of the precessional cycle.

Precession of 50" meant a deduction of 300 breaths (5/6 of a breath/solar day) in fixing the zero point and the cumulative decrement in a precessional cycle was 300x25920 when the true sidereal year got considered. Sexagesimal degree-minute-seconds arc units thus facilitated accounting of precession as 50/60 degree or 50' in 60 years and 1 degree or 60' in 72 years. Thus it is easy to understand that nature itself displays a sexagesimal system by way of the rate of precession of 50" or 360⁰ in 25920 years when the ecliptic is divided into 360⁰/21600'/1296000" and the sexagesimal notation is adopted. With an extra-long sidereal year demanding a deduction of 360 breaths, the precessional cycle for the computed Sun would have been 21600 years.

Computational instrument of Siddhāntas was a model of time consisting of the Yugas. Basic unit of this model was the 21600 years – a concept borrowed from Yogaśāstra as explained above. *Kālapuruṣa of the Siddhāntas had a day of 21600 years as each year was an exhalation-inhalation sequence beginning with Makara-samkrānti.* It's this synonymy with breath that made the samkrāntis sacred occasions and the frame of observance was sidereal zodiac of Siddhāntas. With tropical zodiac synonymy as above was impossible with 21600 years. Extra-long year was a necessity to incorporate a full rotation of seasonal cycle in 21600 years or a day of Kālapuruṣa.

What is said above can be established astronomically using the siddhāntic length of year that is almost anomalistic. Simple arithmetic tells us that –

21600 siddhāntic years of 365.25875 = 21601 tropical years of 365.242 days

i.e. 21600 siddhānta years = 21601 seasonal years (cycles of seasons)

It is therefore evident that the basis of Siddhāntic astronomy was a precession cycle of 21600 years over an anomalistic zodiac requiring annual precession correction of the order of 9 seconds of arc in a year. This leads to 30 revolutions of the stellar cage in a caturyuga of 432,000,0 years, the same as that we find mentioned in Sūryasiddhānta:

त्रिंशत्कृत्वा युगे भानांचक्रं प्राक्परिवर्तते (Sūryasiddhānta: III.9)²⁰

“Wheel of stars make 30 revolutions to the east in a Yuga (of 432,000,0 years)”

In fact with the year length exceeding the true sidereal value, the initial point advanced towards east by nine seconds of arc in a year and this became 54⁰ in 21600 years and 30 revolutions in 432,000.0 years. Śloka means *30 revolutions of the initial point eastward* very clearly but as the basis of such an interpretation could not be understood, a number of misinterpretations were invented to suit the known phenomena. Parameśvarācārya has discussed the verse with his interpretation aimed to suit the precession phenomenon as known to him.

Sūryasiddhānta attests this fact when we note that –

1. Mean sun of Kaliyugādi (17/18 February 3102 BC) was nearly 301⁰
2. Precession arc between the tropical and the siddhāntic zodiac was nearly 50⁰ in 3600 years. For the year length of Sūryasiddhānta, the forward movement of the zero point in 3600 years was nearly 9⁰ as mentioned above. Above factors decided the zero in AD 500 as 301+50+9 = 360 (coinciding the zero point)
3. Precession correction to the zero point was negligible as long as the sidereal zodiac of fixed zero was in use in siddhāntic astronomy. But underwent drastic change with the adoption of vernal equinox as a reference point – annual correction required increased to nearly fifty seconds (50’’) from (-)9 seconds – necessitated a change of one minute or 60’’ in siddhāntic computations.

4. We can see the above confusion, ambiguity in respect of ayanāmśa in the siddhāntas of the period since Āryabhaṭa and Varāhamihira to the times of Mañjula, Bhāskara-I, Haridatta etc. Āryabhaṭa is silent, Mihira speaks of precession in Pañcasiddhāntikā but he is silent about ayanāmśa, Bhāskara is silent and *only Mañjula had the conviction to redefine the value as nearly one minute of arc with reference to vernal equinox.*
5. In AD 500, the Siddhānta Sun and Equinox coincided at 0^0 . Sūryasiddhānta computations imagine zero at Kaliyugādi based on the 54-degree progression of zero point in 21600 years or 9 degree in 3600 years. Going back by 3600 years, in 3102 BC epoch we can see Sun at $360^0 - 9^0 = 351^0, 59^0$ west of the Yugādi equinox. No other precession cycle bridges the computation across 3600 years so well.

Further, we may note that the smallest Yuga is the minimum period in which the zero point progresses eastward over complete revolutions. 54^0 in 21600 years means 360^0 in 6.666 days of Kālapuruṣa. To avoid the fraction of day in a Yuga, the Kaliyuga was defined as 3 times 6.666 or 20 days = 432000 years = 3 revolutions of the zero eastward. This meant 30 revolutions or 200 days of Kālapuruṣa constituted a mahāyuga. Multiples of the basic unit of 21600 was used conveniently in deriving the mean longitudes by framing up the concept of Yugas.

Above discussion, satisfactorily answers the conceptual basis of the Siddhāntic sidereal zodiac of India and the ancient zodiac prevalent in Babylon. Computation was relative to sun and was based on synodic periods. Model conceived the precession cycle as 21600 years, each year representing a breath of the cosmic Man and the precepts on solstices and equinoxes as available in Siddhāntic treatises cannot be fully understood without this basic conception.

Evidence of the Weekdays and a Possible Time Structure of Breathing

We saw above that the sexagesimal system and the 21600' gave to the Sumerians a feel of the Cosmos in numbers. If we examine the rationale of hours and weekdays which the modern civilizations owe to the Sumerians, it can be understood that the notions arose out of the experienced time structure of breathing. Planetary lordship vis-a-vis zodiacal symbolism associated with hours in the order of Saturn, Jupiter, Mars, Sun, Venus, Mercury, and Moon, which repeat in hour cycles point towards a time structure of breathing that inspired the unit of hour as 1/24 of the day. Rationale is evident in the yoga precept –

सार्धद्विघटिके ज्ञेया शुक्लेकृष्णे शशी रविः।
 वहत्येकदिनेनैव यथा षष्टिघटि क्रमात्।
 शुक्लपक्षे भवेत्वामा कृष्णपक्षे च दक्षिणा।
 जानीयात्प्रतिपत्पूर्वं योगी तद्यतमानसः।

“Breath a of Moon or left nostril in light fortnight and breath of Sun or right nostril in dark fortnight covers the 24 hours of the day, alternately for one hour. Yogis should

keenly understand the onset of light and dark fortnights by the occurrence of the breath in the left and right nostrils respectively”

It is therefore apparent that the observation of breathing and counting may have played an important role in ancient civilization as an activity that enabled them to have precise measurement of time. It's what may be described as the 'sexagesimal association' to the diurnal rotation of Earth gave us time Units which reflect a time structure of breathing.

Conclusions:

1. Modern scientific world is yet to reach a consensus on the origin of the sexagesimal system. Review presented has brought out the deficiencies in the prevailing explanations.
2. The 60 base system and the 21600' of the heavenly circuit of Sun is shown to be the outcome of a time reckoning based on the number of breaths. Indian astronomical texts like Sūryasiddhānta and Aryabhatiya have preserved this notion.
3. Ecliptic divided into 21600' facilitated the computation of precession in terms of the mystic 50/60 relationship and this led to the different solar year lengths seen in cuneiform records.
4. Number of breaths as 21600 vis-a-vis the diurnal rotation and the Earth's revolution round the sun in a year in relation to sidereal and tropical years suggest a Sumerian experience of the Cosmos through the application of sexagesimal system to astronomy.
5. It is easy to understand that the mysticism associated with numbers like 50 and 60 and hours, weekdays, the conceptions of the microcosm and macrocosm and their equivalence etc arose out of the Sumerian experience of the time structure of breathing which inspired a sexagesimal notation. .
6. More studies are required to bring out a detailed presentation on the Sumerian basis of Indian astronomy and Yoga.

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